GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

[x] ORIGINAL [ ] REVISION NO. ______

Project No. E-16-625 GTRI/GIT DATE 2/7/83

Project Director: James C. Wu School/Lab AE

Sponsor: Office of Naval Research
Arlington, VA 22217

Type Agreement: SFRC No. N00014-83-K-0106

Award Period: From 1/1/83 To 9/30/83 (Performance) ______ (Reports)

Sponsor Amount: Total Estimated: $30,000 Funded: $ 30,000

Cost Sharing Amount: $ ______ Cost Sharing No: ______

Title: Unsteady Viscous Flow

ADMINISTRATIVE DATA OCA Contact John W. Burdette x4820

1) Sponsor Technical Contact:

2) Sponsor Admin/Contractual Matters:
ONR RR
Ga. Tech
Attn: Thomas A. Bryant
Phone: (404) 881-4213

Defense Priority Rating: NA

Military Security Classification: Unclassified
(or) Company/Industrial Proprietary: NA

RESTRICTIONS

See Attached SFRC Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval – Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with None proposed

COMMENTS:

COPIES TO:

Research Administrative Network	Research Security Services	Research Communications (2)
Research Property Management	Reports Coordinator (OCA)	Project File
Accounting	GTRI	Other ______
Procurement/EES Supply Services	Library	Other

GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 11/21/86

Project No. E-16-625 School/~~Lab~~ AE

Includes Subproject No.(s) N/A

Project Director(s) J. C. Wu GTRC / ~~GIT~~

Sponsor Office of Naval Research

Title Unsteady Viscous Flow

Effective Completion Date: 9/30/83 (Performance) ______ (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [x] Final Invoice or Final Fiscal Report
- [x] Closing Documents
- [ ] Final Report of Inventions - already submitted
- [x] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other ______

Continues Project No. ______ Continued by Project No. ______

COPIES TO:

Project Director
Research Administrative Network
Research Property Management
Accounting
Procurement/GTRI Supply Services
Research Security Services
~~Reports Coordinator (OCA)~~
Legal Services

Library
GTRC
Research Communications (2)
Project File
Other I. Newton
A. Jones
R. Embry

E-16-625

ONR Contract No. N00014-83-K-0106
(Continuation of N00014-75-C-0249)

UNSTEADY VISCOUS FLOW

Final Report

Prepared by

J. C. Wu
Professor and Principal Investigator
School of Aerospace Engineering
Georgia Institute of Technology

for

The Fluid Dynamics Program
Office of Naval Research
800 North Quincy Street
Arlington, Virginia 22217

January 1986

1. INTRODUCTION

This report summarizes a research program carried out by the principal investigator and his graduate students at the Georgia Institute of Technology under the support of the Office of Naval Research in the area of unsteady viscous flows. The reporting period is rather extensive; so is the scope of research carried out by the researchers. Research results and conclusions have been published in a series of reports, Ph.D. theses, conference proceeding papers, journal articles, and chapters in specialist books. In this report, chronological lists of major publications and Ph.D. theses are provided together with a brief description of the research accomplishments that, in the opinion of the principal investigator, represent new and important contributions to the subject of unsteady viscous flows.

The present research program encompasses a theoretical aspect and a computational aspect. Throughout the tenure of this research, these two aspects have been made to reinforce one another. The close interplay of the two aspects has led to a much improved understanding of the physical processes important to the flow of viscous fluid and of the various viscous mechanisms of generating aerodynamic forces. Based on this improved understanding, new numerical approaches for computing viscous flows as well as new theories of aerodynamics dealing with viscous fluids have been conceived, developed, calibrated, and utilized in solving various problems of practical interest. It has been demonstrated conclusively through analyses, numerical illustrations, and theoretical solutions to a variety of problems that the numerical approaches and theories developed under the present research have a broad range of applications. In particular, the numerical approaches and theories are ideally suited for studies of high Reynolds number viscous flow problems, including problems where massive flow separations are present. As is well known, high Reynolds number unsteady separated flows are formidably difficult to treat either theoretically or computationally. The progress made under the present research program is particularly rewarding to the present researchers in that it clearly has opened several new doorways to this extremely important area of fluid mechanics.

In reviewing the work carried out under the support of ONR, the principal investigator recollected vividly the support and encouragement of Morton Cooper of ONR. From the beginning of this program in 1975 until his retirement, Morton provided continually the needed mix of interaction and latitude for the present researchers to pursue a subject which, because of its scope, its formidable difficulties and its fundamental nature, requires long-termed efforts at a vigorous but unharried pace. It appears that, with current increasing emphases on "focused research projects", opportunities for long-termed basic research such as the present one are becoming rarer than before. For this reason, the research environment provided for the present project is very much treasured by university researchers, this principal investigator included. It is thus with great satisfaction as well as a degree of nostalgia that this principal investigator carries out this final phase of effort under ONR support, reporting the work performed under this project.

The research performed under ONR support are centered upon time-dependent laminar incompressible flows past the exterior of solid bodies. This type of flow possesses the essential features of interest and serves to bring into focus the most important concepts associated with the dynamics and aerodynamics of viscous flows. Generalization of these concepts for steady flows, turbulent flows and compressible flows has been undertaken by the principal investigator. The scope of work carried out for these other types of flows, however, is not as extensive as that for time-dependent laminar incompressible flows.

At the conclusion of the research phase of this project, the new numerical and theoretical approaches for unsteady viscous flows were firmly established and it was anticipated that additional efforts of reasonable scope would produce significant improvements in the understanding of unsteady flows, including those containing massive separated regions. It was decided then to carry out some of the needed efforts at no additional costs to ONR and to include the additional results in a final report. This has been accomplished. The originally estimated time span for the additional effort, however, turned out to be totally inadequate because of the new and competing activities taken on by the principal investigator. For this reason, the completion of the present report is very much delayed.

2. SCOPE OF RESEARCH

In the 1960s, the aerodynamics community became fascinated with the potentials of computational fluid dynamics which was in its infancy at the time. A number of researchers in fluid dynamics believed that the advent of high speed digital computers would eventually make it feasible to simulate all types of flows routinely, economically, and accurately. Several major limitations of the theoretical and experimental branches of aerodynamics, well-known to aerodynamic researchers, were expected to be removed by the developing computational branch of aerodynamics.

In late 1960s and early 1970s, remarkable progress were made in the computation of many types of boundary layer and potential flow problems. Problems of high Reynolds number general viscous flows, i.e., flows containing appreciable regions of viscous separations, however, remained mostly beyond the capability of computational fluid dynamics. In fact, in the early 1970s, a number of experienced and active researchers in computational fluid dynamics expressed pessimism regarding the prospect of computing such flows in the foreseeable future. These researchers pointed out the existence of a number of formidable difficulties associated with the computation of general viscous flows and predicted that accurate computations of high Reynolds number viscous flow involving massive flow separations are not feasible in the foreseeable future.

During the early 1970s, the principal investigator established the underlying theories for a new numerical approach to flow problems based on the concept of fundamental solutions of differential equations. The present research was initially proposed for the development of this new approach and for the establishment of a computational capability for computing laminar flowfields around oscillating airfoils at moderate Reynolds numbers (~1000). This original objective was accomplished quickly during the initial phase of this research program. Results of the initial research brought into focus several promising methods for extending the computational capability to high Reynolds number flows. In addition, a general theory for viscous aerodynamics was conceived. Accordingly, in subsequent proposals for continued support by ONR, the scope of the research was broadened. Additional support was provided by other governmental and industrial organizations for the present research. The overall objective of the present research program is briefly as follows.

"To fully develop a general aerodynamic theory and a computational approach well-suited for the study of general viscous flows. To obtain numerical and theoretical results that contribute to a better understanding of various aerodynamic phenomena in steady and unsteady general viscous flows."

Throughout the duration of the present program, unsteady laminar incompressible flows pertinent to aerodynamic applications were the focal problem investigated. During the final stage of the program, however, turbulent flows and compressible flows were also studied. As it is well-known, the problem of incompressible viscous flow is at the center of fluid dynamics by virtue of its fundamental nature and its practical importance. Work related to incompressible unsteady viscous flows in two-dimensional configurations has reached a reasonable stage of completion under the present project. Current efforts dealing with three-dimensional and compressible aspects of unsteady viscous flows are being carried out under different sponsorships.

3. WORK ACCOMPLISHED

3.1. Viscous and Inviscid Flows

The computational and theoretical approaches developed under the present project are based on the concept of vorticity, defined as the curl of the velocity field. The anatomy of unsteady viscous external flows, including those containing large separated regions, is explained under the present research by examining the dynamics of the vorticity field in a viscous fluid. The linkage between the flow of a real, i.e., viscous, fluid and the flow of an idealized "inviscid fluid" is unravelled.

Consider a finite solid body immersed in an infinite incompressible fluid with uniform viscosity. The solid body is at rest initially in a fluid which is also at rest initially. Subsequent motions of the solid induce a corresponding unsteady motion of the fluid. It has been shown that vorticity is neither created nor destroyed in the interior of the fluid domain. Vorticity, however, is continually being generated at the solid boundary in contact with the fluid following the initiation of the solid motion. This vorticity spreads into the interior of the fluid by the process of viscous diffusion and, once there, is transported away from the solid surface by both convection and diffusion. Since the transport of vorticity by convection is a finite rate process and that by diffusion is effectively finite rate, the vortical region of the flow is of finite extent at any finite time level after the initiation of the solid motion. Outside the vortical region, the flow is irrotational and therefore inviscid. If the flow Reynolds number is not small, then the effective rate of viscous diffusion is much smaller than that of convection. Therefore, a large region of the fluid, ahead and to the side of the solid, is free of vorticity and is inviscid.

The general pattern of unsteady flow development can be briefly described as follows. As a consequence of the solid motion relative to the fluid, vorticity is generated continually at the fluid/solid interface. Once generated, the vorticity moves along the solid surface as long as the flow remains attached. That is, since the effective rate of viscous diffusion is much smaller than that of convection, the vorticity, generated on the solid surface, cannot penetrate far into the interior of the fluid domain before being carried downstream by the fluid motion. A thin layer of vorticity adjacent tot he solid boundary is therefore present. This layer is simply the well-known boundary layer. The vorticity within the boundary layer continually moves downstream with the fluid and, at the same time, is continually being replenished through the generation of vorticity on the solid surface. This process of replenishment is present in both steady and unsteady flows. In steady flows, the replenishment process and the vorticity transport process balance one another and the vorticity distribution in the boundary layer is independent of time in a reference frame attached to the solid. In unsteady flows, the replenishment and transport processes do not balance one another and vorticity distribution in the boundary layer is time-dependent. In both steady and unsteady flows, because the boundary layers are thin, it is often convenient to represent the vorticity in the layers by vortex sheets.

The representation of a boundary layer by a vortex sheet does not imply an inviscid fluid assumption. Rather, this representation approximates the location of the vorticity across the boundary layer by a given point adjacent to the solid surface. The strength of the concentrated vortex sheet is simply the integrated vorticity across the boundary layer. The vortex sheet moves along the solid surface. The distinction between the inviscid assumption and the present approximation is not merely a matter of semantics. While the two concepts often lead to the same conveniences in analyses and computation, the present approximation is based on a viscous flow viewpoint and experiences no conceptual difficulties associated with previous inviscid theories.

The vorticity in the boundary layer eventually leaves the vicinity of the solid surface through several possible avenues. If no massive separation of the flow occurs on the solid surface, then the vorticity in the boundary layer eventually feeds into a wake layer. This occurs, for example, in the case of a thin airfoil at a small angle of attack. The two boundary layers at the two sides of the airfoil in this case merge at the trailing edge, with both layers feeding vorticity into the wake layer. In steady flows, the total flux of vorticity entering the wake is zero. In unsteady flows, a net flux of vorticity enters the wake. Since the boundary layers are thin, the wake layer, which is a continuation of the boundary layers, is also thin initially. As the vorticity layer moves away from the solid through the convective process, viscous diffusion produces only a slow growth in the thickness of the wake layer. In consequence, it is reasonable in many applications to represent the wake layer also by a vortex sheet. The wake layer is usually unstable. The velocity field associated wit the vorticity in the wake layer causes the wake layer to "roll-up". If the roll-up process occurs at a large distance from the solid, then it is reasonable to represent the rolled-up vorticity by a single vortex filament in analyzing the flow near the solid. if the roll-up process occurs near the solid, however, then detailed structure of the rolled-up vorticity may be necessary. In any event, the total strength of the rolled-up vorticity needs to be known in order to determine correctly the aerodynamic forces acting on the solid. For three-dimensional flows, the vorticity in the boundary layer leaves the vicinity of the solid surfaces also through the formation of tip vortices which usually roll up.

In applications where flow separation is an important feature, the representation of the vorticity in the boundary layer part of the flow by a vortex sheet is still permissible. Quantitatively accurate solution to the flow problem in this case requires a knowledge not only of the strength of the vortex sheet representing the boundary layer but also of the detailed vorticity distribution in the separated (recirculating) part of the flow. This distributed vorticity is not accurately represented by concentrated vortex sheets or vortex filaments. it is well known, however, that in unsteady flows vortex assemblies often move more or less as an entity. In consequence, considerable physical insight can be gained through a vortex sheet/filament representation even in cases of flows containing massive separated regions.

3.2 Aerodynamic Theory for Viscous Flows

Aerodynamic forces and moments acting on solid bodies immersed and moving in viscous fluids can be determined, in principle, through a quantitative knowledge of the detailed fluid motion around the bodies. The acquisition of detailed information about the real flowfield associated with lifting surfaces, however, presents immense, often insurmountable, mathematical and experimental difficulties. Historically, therefore, the most remarkable advances in aerodynamics were brought about by aerodynamicists who perceived approaches for the prediction of aerodynamic forces and moments that avoid, as much as possible, entanglement with the details of the fluid motion. In particular, the circulation theory is known to predict the lift force accurately for certain types of lifting surfaces, e.g., thin airfoils with sharp trailing edges, under certain flow environments, e.g., small angles of attack.

The circulation theory was until recently the foundation of accepted theories of aerodynamics. Considerable uncertainties and conceptual difficulties, however, existed regarding the application of the circulation theory in cases where the lifting surface does not possess a sharp trailing edge or where more than one trailing edge is present, where massive flow separation occurs, and where the lifting surface is three-dimensional and its motion is time-dependent. These uncertainties arose mainly because of the perfect-fluid assumption in the mathematical development of the theory. The viscous origin of circulation were long recognized and several well-known works, e.g., by Von Karman and Millikan, by Howarth and by Sears, dealt with certain aspects of viscous phenomena that produce circulation. A thorough understanding of the viscous mechanisms of generation of steady and time-dependent aerodynamic forces, however, was not available and it was usually difficult to interpret the application of the circulation theory as an approximation of the viscous flow phenomena.

Under the present research program, a general theory for aerodynamic forces and moments in viscous flows was rigorously established on the basis of the Navier-Stokes and continuity equations. No simplifying assumptions, other than those contained in the Navier-Stokes equations, were introduced in the derivation of this theory.

The general theory comprises the following three mathematical statements:

$$\int_{R_\infty} \vec{\omega}\, dR = 0 \tag{1}$$

$$\vec{F} = -\frac{\rho}{d-1}\,\frac{d}{dt}\int_{R_\infty} \vec{r} \times \vec{\omega}\, dR + \rho\,\frac{d}{dt}\int_{R_s} \vec{v}\, dR \tag{2}$$

$$\vec{M} = \frac{\rho}{2}\,\frac{d}{dt}\int_{R_\infty} r^2\, \vec{\omega}\, dR + \rho\,\frac{d}{dt}\int_{R_s} \vec{r} \times \vec{v}\, dR \tag{3}$$

where R_∞ is the infinite unlimited region jointly occupied by the fluid and the solid bodies; $\vec{F}$ is the aerodynamic force acting on the solid bodies; d is the dimensionality of the problem, i.e., d = 2 for two-dimensional flows and d = 3 for three-dimensional flows, $\vec{r}$ is a position vector; R_s is the region occupied by the solid bodies, and $\vec{M}$ is the moment of aerodynamic force acting on the solid bodies.

Equations (1), (2) and (3) are valid for the incompressible motion of an infinite fluid with uniform viscosity and with one or more solid bodies immersed in the fluid. The motions are considered to start from rest and are generally time-dependent. Steady flows, when they exist, are considered to be approached asymptotically at large time levels after the onset of the motion. Equation (1) states that the combined total vorticity of the fluid and the solid bodies is zero. The vorticity is defined as the curl of the velocity field. In the solids, the vorticity is simply twice the angular velocity of the solid bodies. In cases where the region occupied by the solid bodies is negligibly small, or where the solid bodies undergo only translational motions, the vorticity in the solid is zero. For these cases, existing inviscid aerodynamic theories correctly require the total circulation of the whole system, including the bound vortex, the starting vortex and the wake vortices, to be zero. The effects of rotation of the solid bodies and of distributed vorticity, correctly given by Eqs. (1), are generally not included in inviscid analyses.

Equation (2) states that the aerodynamic force acting on the solid bodies is composed of two contributions. The first term on the right side of Eq. (2) gives the contribution of the time variation of the first moment of the vorticity. The second term gives the contribution of the inertia force of the fluid displaced by the solid bodies. Equation (3) states that the moment of aerodynamic force is composed of two contributions, a contribution of the total second moment of the vorticity field and a contribution of the moment of inertia. The inertia terms in Eqs. (2) and (3) of course vanish in cases where the solid region is negligibly small or where the solid bodies experience no acceleration.

As discussed earlier, it is convenient to divide the overall unsteady flow problem into its kinetic and kinematic aspects. The general theory described here relates the unsteady aerodynamic forces and moments acting on the solid bodies to the kinetic development of the vorticity field. The task of analyzing the kinetics and the kinematics of the flow remains.

It is clear from the general theory, that all the information about aerodynamic forces and moments are contained in the time-dependent vorticity environment of the lifting body. No information about the potential field surrounding the vortical region is needed in the theory. Under certain restrictive circumstances, it is possible to specify the vorticity field approximately without actually solving the vorticity transport equation. For example, with an attached flow, the strength of the vortex sheet representing the boundary layer vorticity can often be determined analytically. The general theory described above then permits the unsteady aerodynamic forces and moments to be determined in a straightforward manner.

Details of the general theory are presented in Publication #26 in Section 4.1 of this report.

3.3 Zonal Procedure

Like the general viscous theory of aerodynamics, the computational approach developed in the current research program deals with distributed vorticity existing in viscous flows. The general computation procedure is composed of two parts, a kinetic part which determines the time development of the vorticity field in the fluid and a kinematic part which determines the velocity field at each instant of time corresponding to the vorticity field at that instant of time. The anatomy of viscous flows suggested a number of simplifications that are justifiable on physical grounds. It needs to be emphasized again, however, that the simplifications utilized in the present research are not "inviscid assumptions" or "idealizations", although in some cases, the results of the simplifications do appear similar to results of classical inviscid aerodynamic analyses. For example, a thin boundary layer is sometimes justifiably approximated by a vortex sheet. This approximation is justified because of the thinness of boundary layers in high Reynolds number flows. No inviscid assumption is involved, only the precise distribution across the boundary layer is compromised for computational efficiency. This compromise yields accurate results as long as the boundary layers are sufficiently thin, as they generally are in high Reynolds number flows. No conceptual difficulty arises from the approximation.

The computational approach developed under this present research project was further developed after the completion of the present project. The result is a zonal method for the solution of high Reynolds number flow problem which is extremely efficient and accurate. This zonal method is ideally suited for unsteady flow computations. A summary of the present status of development of this method is presented in Publication #45 of Section 4.1.

It is important to note that the zonal procedure is completely compatible with the general viscous theory of aerodynamics described in Section 32. The zonal method and the general theory has in common a number of important attributes. These attributes are:

1) The general theory and the zonal method are general in that they are valid for all incompressible flows of viscous fluids, including those containing massive separated regions.
2) The general theory and the zonal method permit precise aerodynamic computations to be carried out with a knowledge of only the viscous zones.
3) The general theory and the zonal method offers an opportunity to analyze the various time-dependent viscous zones separately. With the zonal method, there is no need to match, iteratively during each time step, the boundary layer zone to the detached viscous zones (the separated zone, the wake zone and the starting vortex zones). With the general theory, the important flow elements contributing to unsteady aerodynamic loads can be identified and the relative importance of each contributor can be assessed.

4) Remarkably simple procedures can be established on the basis of the general theory and the zonal method to analyze and to compute high Reynolds number flows containing no appreciable separated regions. In particular, explicit expressions for the aerodynamic loads are often obtainable and panel/vortex lattice methods can be utilized in accurate computations of the unsteady aerodynamic performance of various aerodynamic surfaces.

5) For flows containing boundary layer as well as separated zones, the attached portion of the viscous flow can still be analyzed in an extremely simple manner. In conjunction with the zonal method, an extremely accurate procedure whose computational requirements are minimal for flows at all Reynolds numbers can be established.

3.4 Theoretical and Computational Results

A large number of unsteady viscous flow problems were studied theoretically and computationally using the approaches developed under the present project. A summary of computed and theoretical results obtained under the present project is included in Publication #34 in Section 4.1 of this report.

4. LIST OF PUBLICATIONS

The list of publications given below resulted from research efforts in the area of unsteady viscous flows carried out by the principal investigator and his co-workers under the support of ONR and of other agencies. In addition to these publications, a list of Ph.D. theses completed in the same area is provided. Several M.S. research papers and internal reports were prepared although a list of these papers and reports are not included in the present final report. In the following lists, tic marks indicate items in which ONR support is acknowledged and asterisks indicate invited articles.

4.1 List of Articles

1. J. C. Wu and J. F. Thompson, "Numerical Solution of Unsteady, Three-Dimensional Navier-Stokes Equations," Proceedings Project SQUID Workshop on Fluid Dynamics of Unsteady, Three-Dimensional, and Separated FLOWS, pp. 253-280, Purdue University, Lafayette, Indiana, October, 1971.

2. J. C. Wu and J. F. Thompson, "Numerical Solutions of Time-Dependent Incompressible Navier-Stokes Equations Using an Integro-Differential Formulation," Vol. 1, No. 2, pp. 197-215, Journal of Computers and Fluids, 1973.

3. J. F. Thompson, S. P. Shanks, and J. C. Wu, "Numerical Solution of the Three-Dimensional Navier-Stokes Equations in Integro-Differential Form: Flow About a Finite Body," Proceedings AIAA Computational Fluid Dynamics Conference, pp. 123-132, July 1973.

4. J. F. Thompson, S. P. Shanks, and J. C. Wu, "Numerical Solution of Three-Dimensional Navier-Stokes Equations Showing Trailing Tip Vortices," AIAA Journal, Vol. 12, No. 6, pp. 787-794, June 1974.

*5. J. C. Wu, "Integral Representations of Field Variables for the Finite Element Solution of Viscous Flow Problems," Proceedings of the 1974 Conference on Finite Element Methods in Engineering, pp. 827-840, Clarendon Press, 1974.

6. J. C. Wu, "Division of Computation Field for the Finite Element Method," Proceedings of International Symposium on Finite Element Method in Flow Problems, pp. 767-770, University of Alabama at Huntsville Press, 1974.

7. J. C. Wu, A. H. Spring, and N. L. Sankar, "A Flowfield Segmentation Method for the Numerical Solution of Viscous Flow Problems," Proceedings of the Fourth International Conference on Numerical Methods in Fluid Dynamics, Lecture Notes in Physics, Vol. 35, pp. 452-457, Springer-Verlag, 1975.

8. J. C. Wu, "Velocity and Extraneous Boundary Conditions of Viscous Flow Problems," AIAA Paper No. 75-47, American Institute of Aeronautics and Astronautics, 1975.

√9. J. C. Wu, "Finite Element Solution of Flow Problems Using Integral Representation," Proceedings of Second International Symposium on Finite Element Methods in Flow Problems, International Centre for Computer Aided Design, Conference Series No. 2/76, pp. 205-216, June 1976.

10. J. C. Wu and S. Sampath, "A Numerical Study of Viscous Flows Around Airfoils," AIAA Paper 76-337, American Institute of Aeronautics and Astronautics, 1976.

√11. J. C. Wu, "Numerical Boundary Conditions for Viscous Flow Problems," AIAA Journal, Vol. 14, No. 8, pp. 1042-1049, 1976.

√12. J. C. Wu and N. L. Sankar, "Explicit Finite Element Solution of the Viscous Flow Problem," Proceedings of the 1976 International Conference on Finite Element Methods in Engineering, University of Adelaide, Australia, 1976.

13. J. C. Wu and M. Wahbah, "Numerical Solution of Viscous Flow Equations Using Integral Representations," Proceedings of the Fifth International Conference on Numerical Methods in Fluid Dynamics, Lecture Series in Physics, Springer-Verlag, Vol. 59, pp. 448-453, 1976.

√*14. J. C. Wu, "Prospects for the Numerical Solution of General Viscous Flow Problems," Proceedings of the Viscous Flow Symposium, LFTTER0044, pp. 39-104, 1976.

15. J. C. Wu and A. Sugavanam, "A Method for the Numerical Solution of Turbulent Flow Problems," AIAA Paper No. 77649, AIAA Journal, Vol. 16, No. 9, pp. 948-955, Sept. 1978.

16. J. C. Wu, M. Wahbah, and A. Sugavanam, "Some Numerical Solutions of Turbulent Flow Problems by the Use of Integral Representations," Proceedings of Symposium on Applications of Computer Methods in Engineering, Vol. 11, pp. 983-992, University of Southern California, 1977.

√17. J. C. Wu, N. L. Sankar and S. Sampath, "A Numerical Study of Unsteady Viscous Flows Around Airfoils," Proceedings of AGARD Symposium on Unsteady Aerodynamics, AGARD-CP-227, pp. 24-1-18, February 1978.

√*18. J. C. Wu, "Prospects for Computational Aerodynamics," Proceedings of NASA Workshop on Future Computer Requirements for Computational Aerodynamics, NASA CP-2032, February 1978.

√19. N. L. Sankar and J. C. Wu, "Viscous Flow Around Oscillating Airfoil A Numerical Study," AIAA Paper 78-1225, American Institute of Aeronautics and Astronautics, 1978.

20. M. M. Wahbah, "Computation of Internal Flows with Arbitrary Boundaries using the Integral Representation Method," Georgia Institute of Technology Peport, March 1978.

√21. J. C. Wu and Y. M. Rizk, "Integral-Representation Approach for Time-Dependent Viscous Flows," Proceedings of the Sixth International Conference on Numerical Methods in Fluid Dynamics, Also Lecture Notes in Physics, Vol. 90, pp. 558-564, Springer-Verlag, 1978.

√22. J. C. Wu, M. M. Wahbah and A. Sugavanam, "Numerical Solution of Unsteady Flow Problems Using Integro-Differential Approach," Proceedings of the ASME Symposium on Nonsteady Fluid Dynamics, December 1978.

√23. J. C. Wu, "A Theory for Aerodynamic Forces and Moments," Georgia Institute of Technology Report, June 1978.

√24. J. C. Wu, J. E. Hackett and D. E. Lilley, "A Generalized Wake-Integral Approach for Drag Determination in Three-Dimensional Flows," AIAA Paper 79-0279, January 1979.

√25. J. C. Wu and U. Gulcat, "Separate Treatment of Attached and Detached Flow Regions in General Viscous Flows," AIAA Journal, Vol. 19, No. 1, pp. 20-27, 1981.

√26. J. C. Wu, "Aerodynamic Force and Moment in Steady and Time-Dependent Viscous Flows," AIAA Journal, Vol. 19, No. 4, pp. 432-441, 1981.

*27. J. C. Wu, "Accommodation of Diverse Length Scales in General Viscous Flows," Innovative Numerical Analysis for the Engineering Sciences, Shaw et al., Editors, pp. 201-212, 1980.

√*28. J. C. Wu, "Integral-Representation Approach for the Numerical Solution of Flow Problems," Finite Elements in Water Resources, edited by S. Y. Wang et al., University of Mississippi, 1980.

29. J. C. Wu, "Lectures in Computational Fluid Dynamics," in Chinese, Peking University, 1980.

√*30. J. C. Wu, "Hybrid Procedures for Computing General Viscous Flows," Proceedings of Symposium on Numerical and Physical Aspects of Aerodynamic Flows, California State University at Long Beach, 1981.

31. J. C. Wu, M. M. El-Refaee and S. G. Lekoudis, "Solutions of the Unsteady Two-Dimensional Compressible Navier-Stokes Equation Using the Integral Representation Method," AIAA Paper 81-0046, 1981.

32. M. M. El-Refaee, J. C. Wu and S. G. Lekoudis, "Solutions of the Compressible Navier-Stokes Equations Using the Integral Method," AIAA Journal, Vol. 20, No. 3, 1981, pp. 356-362.

33. J. C. Wu and U. Gulcat, "A Body-Fitted Conformal Mapping Method with Grid-Spacing Control," NASA CP 2166, pp. 545-560, 1980.

*34. J. C. Wu, "Problems of General Viscous Flows," Chapter 4 of Developments in Boundary Element Methods - 2, Editors: R. Shaw and P. Banerjee, Applied Science Publishers, London, 1982, pp. 69-109.

√*35. J. C. Wu, "Principal Solutions and Finite Element Procedures," Proceeding of the Fourth International Symposium on Finite Element Methods in Flow Problems, Tokyo, Japan, 1982.

√36. J. C. Wu, "Sources, Sinks, Vortices and Flow Computations," Proceedings of International Conference on Finite Element Methods, Shanghai, China, 1982.

37. J. C. Wu, M. M. El-Refaee and S. G. Lekoudis, "Solutions of the Compressible Navier-Stokes Equations Using the Integral Method," AIAA Journal, Vol. 20, No. 3, pp. 356-362, 1982.

38. J. C. Wu and A. Sugavanam, "Numerical Study of Separated Turbulent Flow Over Airfoils," AIAA Journal, Vol. 20, No. 4, pp. 464-470, 1982.

39. J. C. Wu, "Lectures in Computational Aerodynamics," in Chinese, Northwestern Polytechnical University, Xian, China, 1983.

40. J. C. Wu, N. L. Sankar and T. M. Hsu, "Some Applications of a Generalized Aerodynamic Forces and Moment Theory," AIAA Paper No. 83-0543, January 1983.

√*41. J. C. Wu and U. Gulcat, "Zonal Solution of Unsteady Navier-Stokes Problems," Proceedings of the Third International Symposium on Numerical Methods in Engineering, P. Lascaux, Editor, Pluralis, Paris, pp. 137-145, 1983.

√*42. J. C. Wu, Y. M. Rizk and N. L. Sankar, "Problems of Time-Dependent Navier-Stokes Flow," in Developments in Boundary

Element Methods - 3, Editor: P. K. Banerjee and S. Mukherjee, Applied Science Publishers, London, pp. 137-169, 1984.

√*43. J. C. Wu, "Fundamental Solutions and Numerical Methods for Flow Problems," International Journal for Numerical Methods in Fluids, Vol. 4, pp. 185-201, 1984.

44. J. C. Wu, N. L. Sankar and H. Hu Chen, "Theoretical Study of Non-Linear Unsteady Aerodynamics of a Non-Rigid Lifting Body," Proceedings of Workshop on Unsteady Separated Flows, Editors, M. S. Francis and M. W. Luttges, pp. 137-147, 1984.

45. J. C. Wu and C. M. Wang, "Zonal Solution of Unsteady Viscous Flow Problems," AIAA Paper 84-1637, June 1984.

46. J. C. Wu and H. Hu Chen, "Unsteady Aerodynamics of Articulate Lifting Bodies," AIAA Paper 84-2184, 1984.

47. J. C. Wu and C. M. Wang, "Numerical Solutions of Navier-Stokes Problems Using Integral Representations with Series Expansions," AIAA Paper 85-0034, January 1985.

48. J. C. Wu, T. M. Hsu, W. Tang and N. L. Sankar, "Viscous Flow Results for the Vortex-Airfoil Interaction Problem," AIAA Paper 85-4053, October, 1985.

49. J. C. Wu and C. M. Wang, "Unsteady Aerodynamics of Airfoils Oscillating In and Out of Dynamic Stall," AIAA Paper 85-4078, October 1985.

√*50. J. C. Wu, "Vorticity Dynamics - A Viscous Point of View Over Aerodynamics," Third Symposium on Numerical and Physical Aspects of Aerodynamic Flows, January 1985, Springer-Verlag.

*51. J. C. Wu, "Computational and Theoretical Studies of Unsteady Viscous Aerodynamics," Conference on Low Reynolds Number Airfoil Aerodynamics, June 1985.

*52. J. C. Wu, "Boundary Integrals and Numerical Solution of Inhomogeneous Partial Differential Equations," Proc. 7th Int'l Conf. on Boundary Element Methods in Engineering, September 1985.

*53. J. C. Wu, "Boundary Element Methods and Inhomogeneous Parabolic Equations," Proc. Boundary Element Technology Conference, November 1985.

4.2 Ph.D. Theses

1. J. F. Thompson, "Two Approaches to the Three-Dimensional Jet-in-Cross Wind Problem: A Vortex Lattice Model and a Numerical Solution of the Navier-Stokes Equations," 1971.

2. S. Sampath, "A Numerical Study of Incompressible Viscous Flows Around Airfoils," 1977.

√3. N. L. Sankar, "Numerical Study of Laminar Unsteady Flow Over Airfoils," 1977.

√4. A. Sugavanam, "Numerical Study of Separated Turbulent Flow Over Airfoils," 1979.

√5. Y. Rizk, "An Integral-Representation Approach for Time-Dependent Viscous Flows," 1980.

√6. U. Gulcat, "Separate Numerical Treatment of Attached and Detached Flow Regions in General Viscous Flows," 1981.

7. M. M. El-Refaee, "A Numerical Study of Laminar Unsteady Compressible Flow Over Airfoils," 1981.

8. C. M. Wang, "Computation of Steady Internal Flows Using the Integral Representations Method with Finite Series Expansions," 1983.

5. CONCLUDING REMARKS

The present research project contains a theoretical aspect and a computational aspect. Under this project, new computational and theoretical (non-computational) approaches ideally suited for studies of unsteady aerodynamic problems were conceived and explored. These theoretical and computational approaches are useful for flows containing massive separated regions as well as for those involving only attached viscous regions. They have in common two distinguishing attributes:

(1) They permit aerodynamic analyses to be confined to the vortical (viscous) part of the unsteady flow. The potential flow components need not be evaluated. Only the vortical components enter the process of analyses.

(2) They permit the contributions of the several vortical flow components to the aerodynamic load to be identified and evaluated individually. Through this ability, important flow elements contributing to unsteady aerodynamic load can be identified and their relative importance can be assessed.

These two attributes removed the serious theoretical and computational difficulties associated with the simultaneous presence of flow zones with diverse physical and mathematical characteristics. At the conclusion of the present project, some aspects of the above two attributes have been studied. It was conclusively demonstrated that the theoretical and computational approaches are totally compatible to one another and they lead to an unprecedented opportunity of further research in unsteady flows. This opportunity is being used, under separate support, in the study of several important practical problems of unsteady aerodynamics. The development of the combined theoretical-computational approach is now in a reasonable stage of completion for two-dimensional problems. Research into three-dimensional problems has been initiated.